CSIRO

AUT UNIVERSITY
TE WĀNANGA ARONUI O TAMAKI MAKAU RAU

ASTRON

Cramming CMACs into DSPs

Norbert Abel, Will Kamp
High Performance Computing Research Lab
Auckland University of Technology, New Zealand

Gianni Comoretto
Arcetri Astrophysical Observatory, Italy

15th February 2019 - C4SKA @ AUT

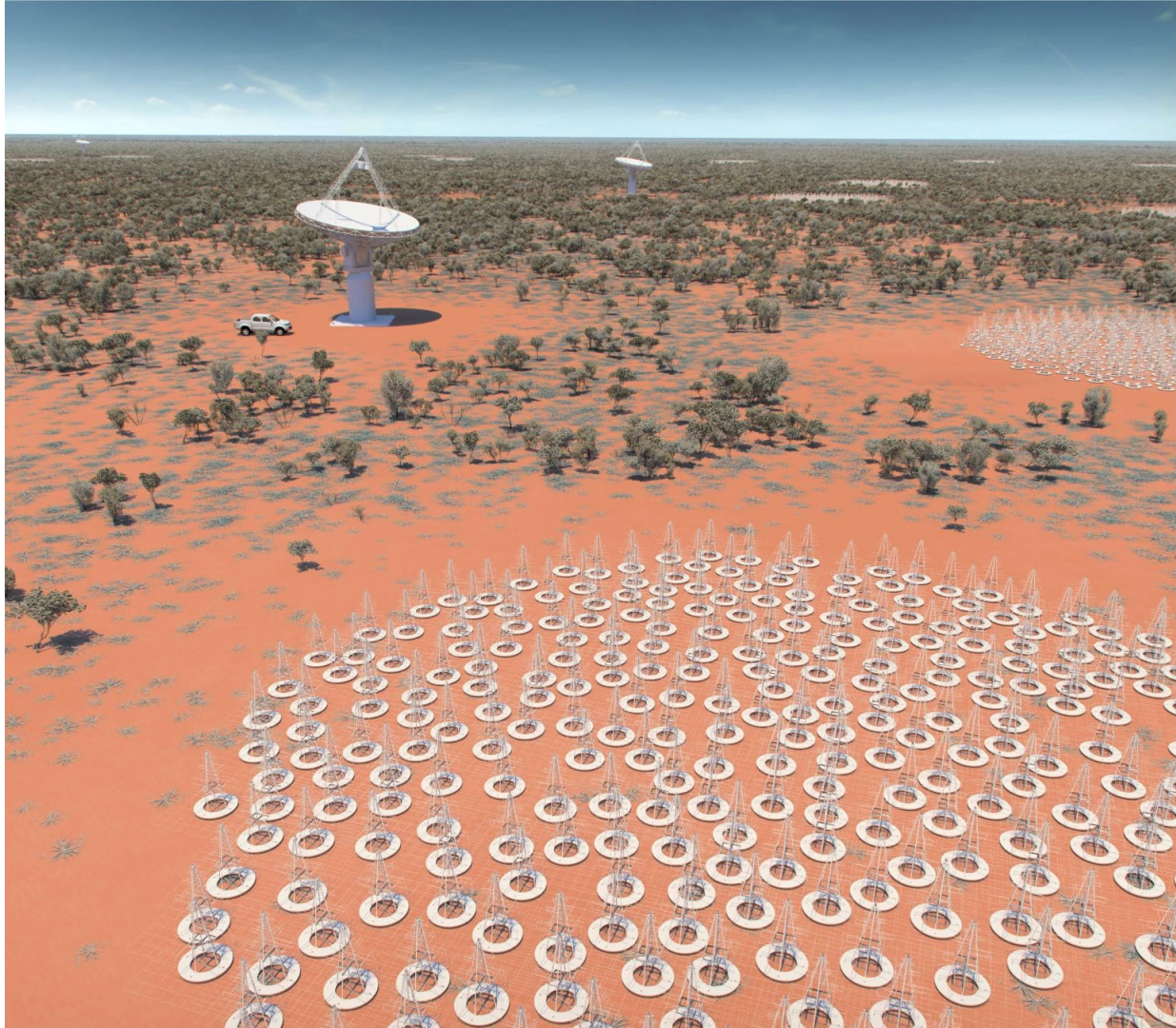
perentie
SKA Low Correlator & Beamformer

Published as *"Complex Multiply Accumulate Cells for the Square Kilometre Array Correlators"* at the *ReConfig 2018* in Cancun, Mexico
Won *"Best Paper Award"*

SKA1 - Low

512 stations
(dual polarisation)

256 x 40 Gbps optical fibre links back to the Central Signal Processor (CSP)



CSP – Central Signal Processor

perentie
SKA Low Correlator & Beamformer

Cross-multiply every station with every other station: $O(N^2)$ operations.
Where N = 2*512 = 1024 for Low.
Then accumulate to “beat down the noise” by the Central Limit Theorem.

Largely implemented with FPGA clusters: 288 Xilinx Ultrascale+ VU37P

SKA.Low = 288 FPGA x 1056 *CMAC* units @ 533 MHz

Complex Multiplication

perentie
SKA Low Correlator & Beamformer

Cartesian Multiplication:

$$z = x \cdot y$$
$$= (a + ib) \cdot (c + id)$$
$$= (a \cdot c - b \cdot d) + i(a \cdot d + b \cdot c)$$

4 Multiplies, 2 Additions/Subtractions

Karatsuba (Gaussian) Multiplication:

$$K_1 = a \cdot c$$
$$K_2 = b \cdot d$$
$$K_3 = (a + b) \cdot (c + d) = (a \cdot c + b \cdot c + a \cdot d + b \cdot d)$$
$$z = (K_1 - K_2) + i(K_3 - K_1 - K_2)$$

3 Multiplies, 5 Additions/Subtractions

Complex Multiplication

Cartesian Multiplication:

$z = x \cdot y$

$= (a + ib) \cdot (c + id)$

$= (a \cdot c - b \cdot d) + i(a \cdot d + b \cdot c)$

4 Multiplies, 2 Additions/Subtractions

Karatsuba (Gaussian) Multiplication:

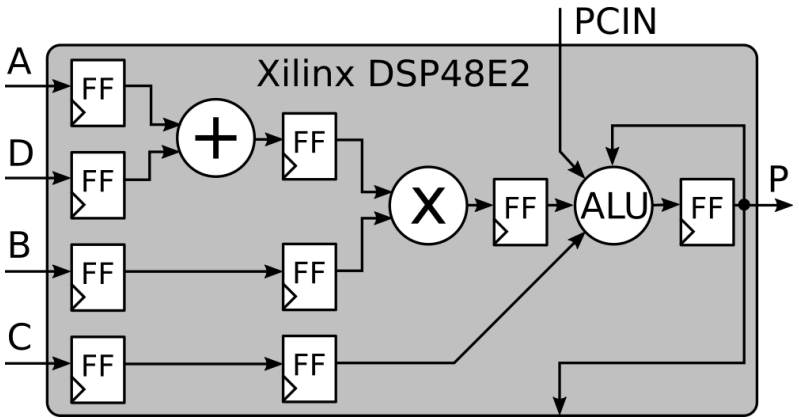
$K_1 = a \cdot c$

$K_2 = b \cdot d$

$K_3 = (a + b) \cdot (c + d) = (a \cdot c + b \cdot c + a \cdot d + b \cdot d)$

$z = (K_1 - K_2) + i(K_3 - K_1 - K_2)$

3 Multiplies, 5 Additions/Subtractions



Xilinx Ultrascale+
One 18x27b signed multiply.
=> 3168 DSPs

Complex Multiplication

Cartesian Multiplication:

$z = x \cdot y$

$= (a + ib) \cdot (c + id)$

$= (a \cdot c - b \cdot d) + i(a \cdot d + b \cdot c)$

4 Multiplies, 2 Additions/Subtractions

Karatsuba (Gaussian) Multiplication:

$K_1 = a \cdot c$

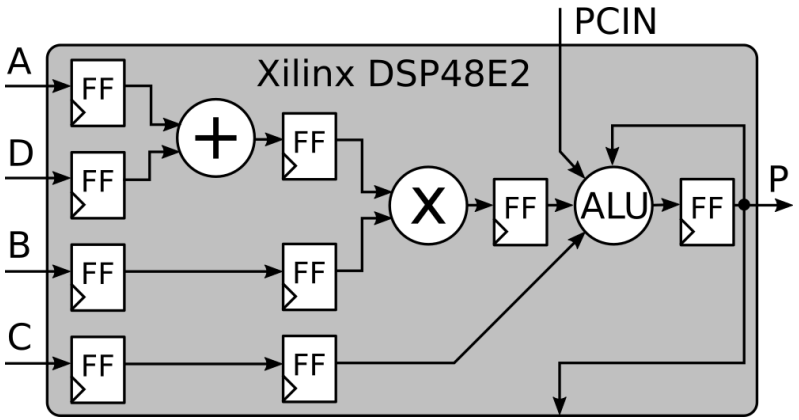
$K_2 = b \cdot d$

$K_3 = (a + b) \cdot (c + d) = (a \cdot c + b \cdot c + a \cdot d + b \cdot d)$

$z = (K_1 - K_2) + i(K_3 - K_1 - K_2)$

3 Multiplies, 5 Additions/Subtractions

Actually, we only need a 8b · 8b multiplier!



Xilinx Ultrascale+
One 18x27b signed multiply.
=> 3168 DSPs

The cramming begins...

$x = a + ib \Rightarrow x' = a + 2^{2w}b$

$y = c + id \Rightarrow y' = c + 2^{2w}d$

$z' = x' \cdot y'$

$z' = (a + 2^{2w}b) \cdot (c + 2^{2w}d)$

$z' = 2^{4w}b \cdot d + 2^{2w}b \cdot c + 2^{2w}a \cdot d + a \cdot c$

$z' = 2^{4w}b \cdot d + 2^{2w}(b \cdot c + a \cdot d) + a \cdot c$

	b	0	0
+	0	0	a
x' =	b	0	a

	d	0	0
+	0	0	c
y' =	d	0	c

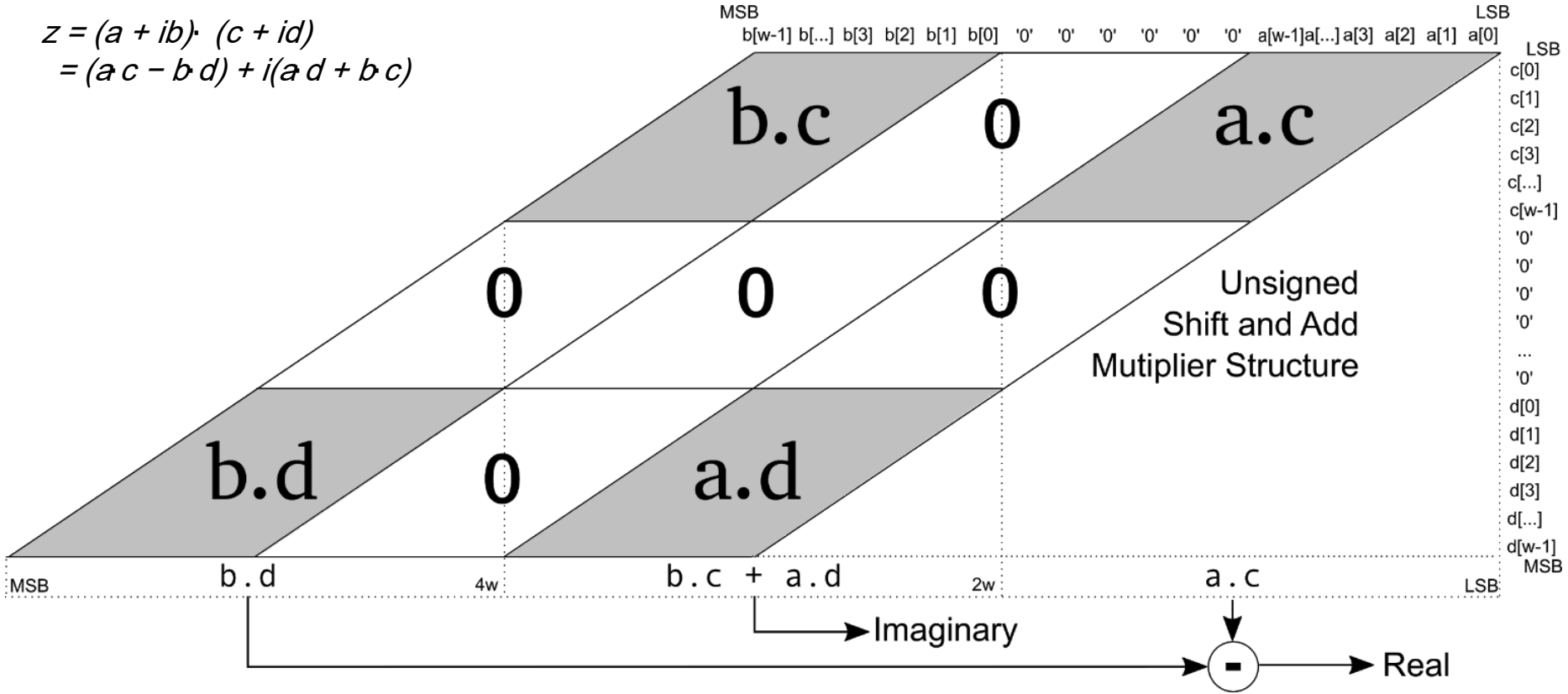
z' =	b·d	b·c + a·d	a·c

$z = b \cdot d - a \cdot c + i(b \cdot c + a \cdot d)$

Unsigned Complex Multiplication

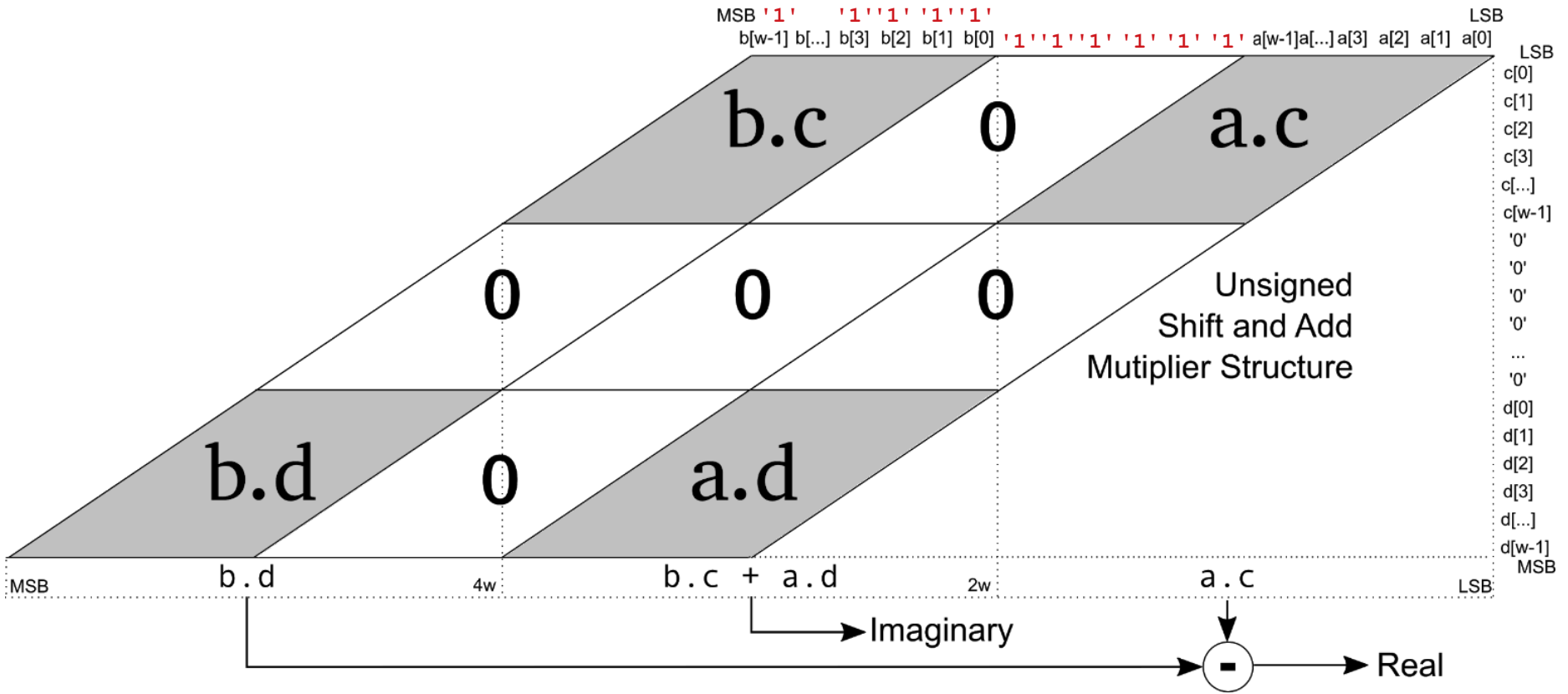
perentie
SKA Low Correlator & Beamformer

$$z = (a + ib) \cdot (c + id)$$
$$= (a \cdot c - b \cdot d) + i(a \cdot d + b \cdot c)$$



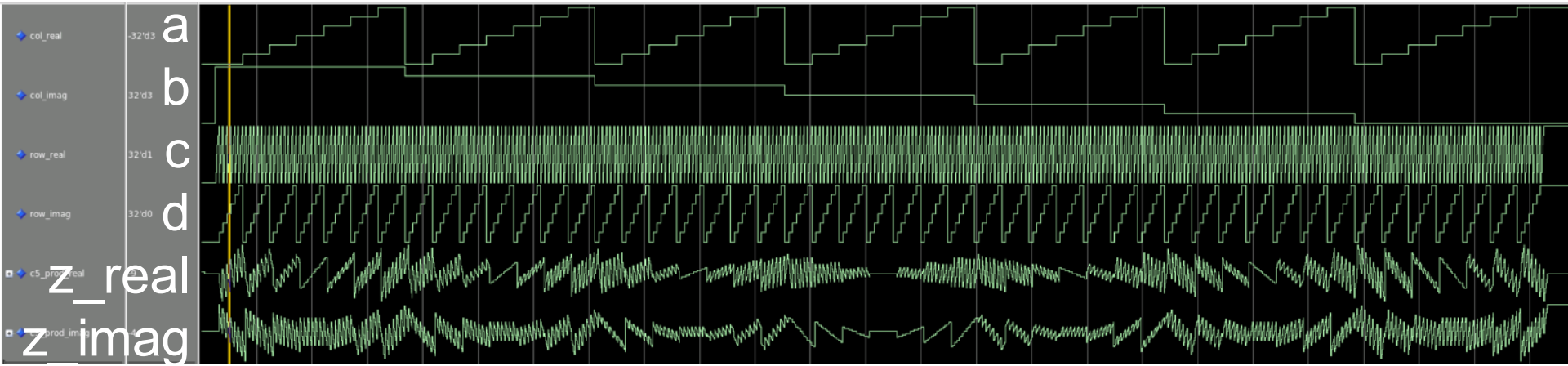
Signed Complex Multiplication

perentie
SKA Low Correlator & Beamformer



Signed Complex Multiplication

perentie
SKA Low Correlator & Beamformer

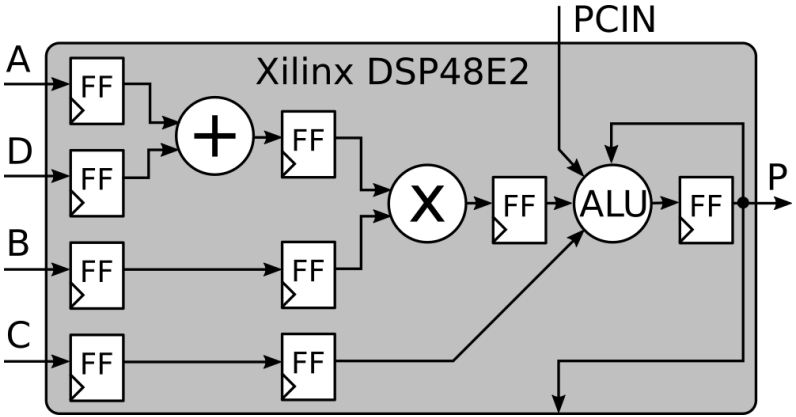


Simulation for w=3, therefore a,b,c,d in range -3 to +3.

It works! Except for the one case where $a = b = c = d = 2^w$

- but the SKA uses -2^w as NaN anyway

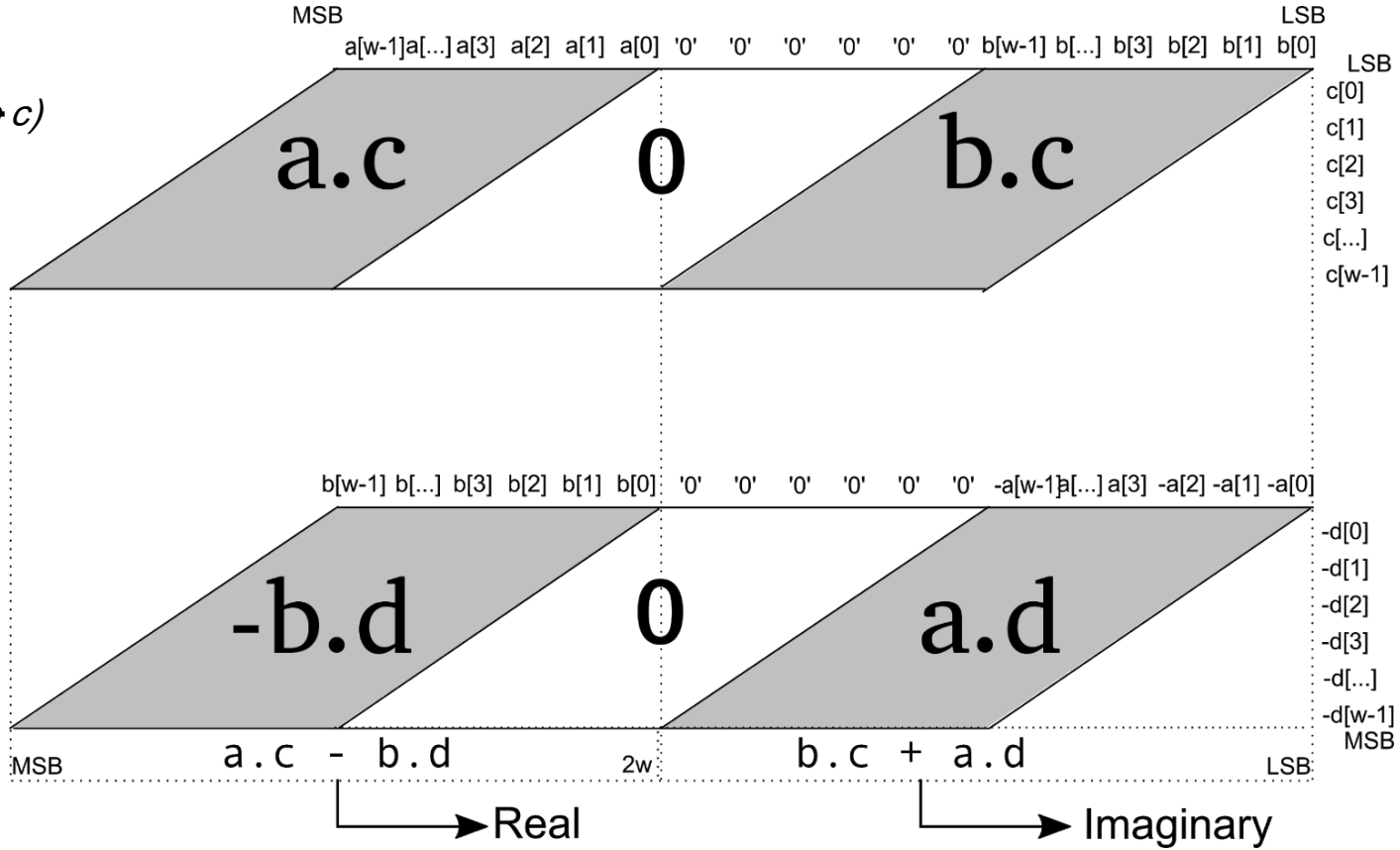
Ultrascale+ DSPs



Xilinx Ultrascale+
One 18x27b signed multiply.

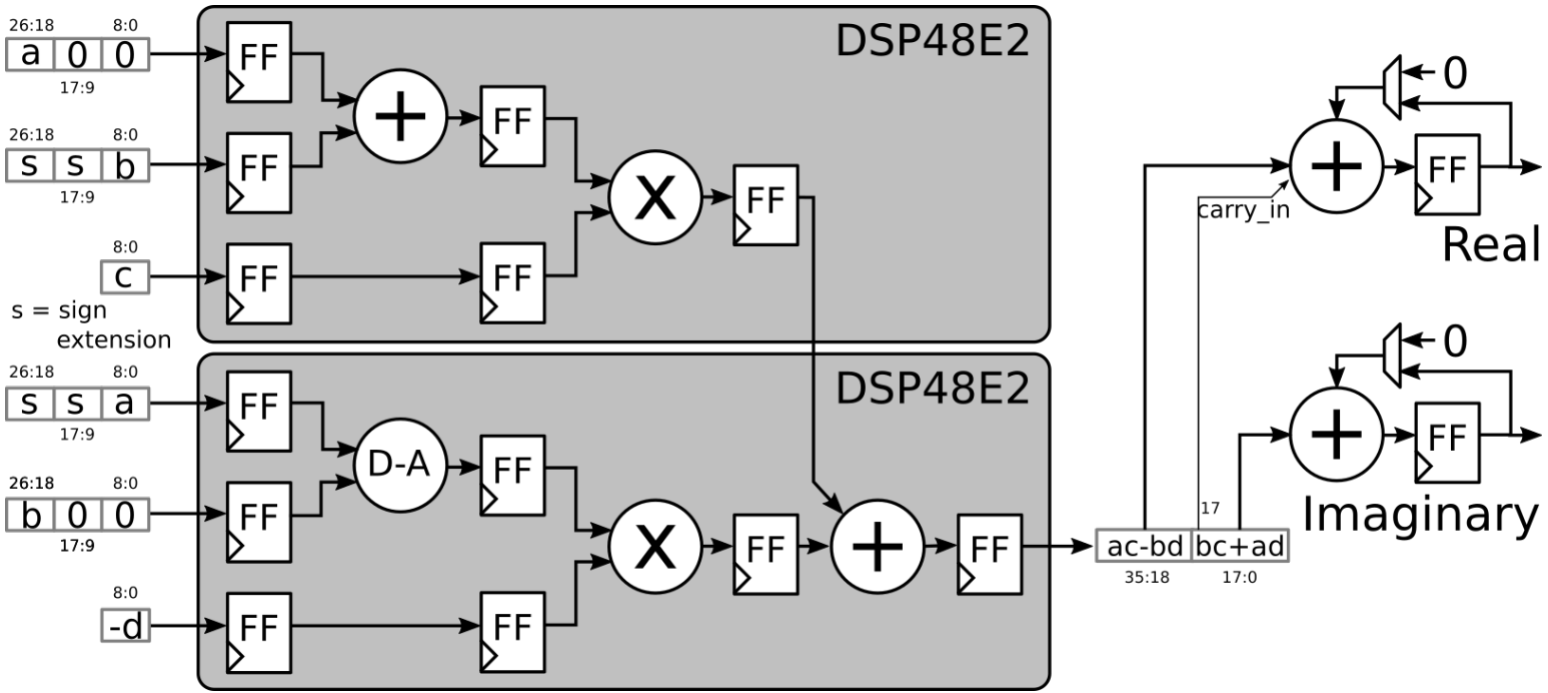
Ultrascale+ DSPs

$$z = (a + ib) \cdot (c + id)$$
$$= (a \cdot c - b \cdot d) + i(a \cdot d + b \cdot c)$$



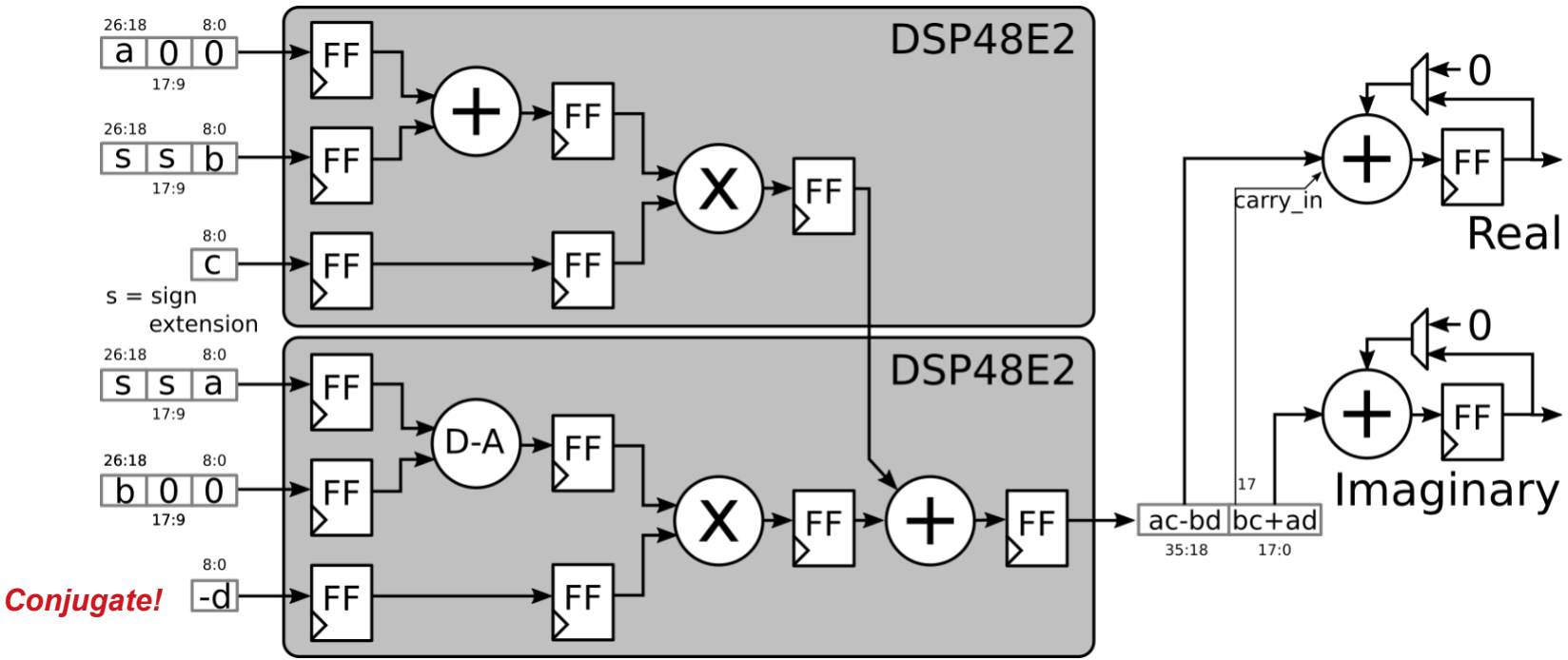
Ultrascale+ DSPs

$z = (a + ib) \cdot (c + id) = (ac - b \cdot d) + i(a \cdot d + b \cdot c)$



Ultrascale+ DSPs

$z = (a + ib) \cdot (c + id) = (ac - b \cdot d) + i(a \cdot d + b \cdot c)$



Results

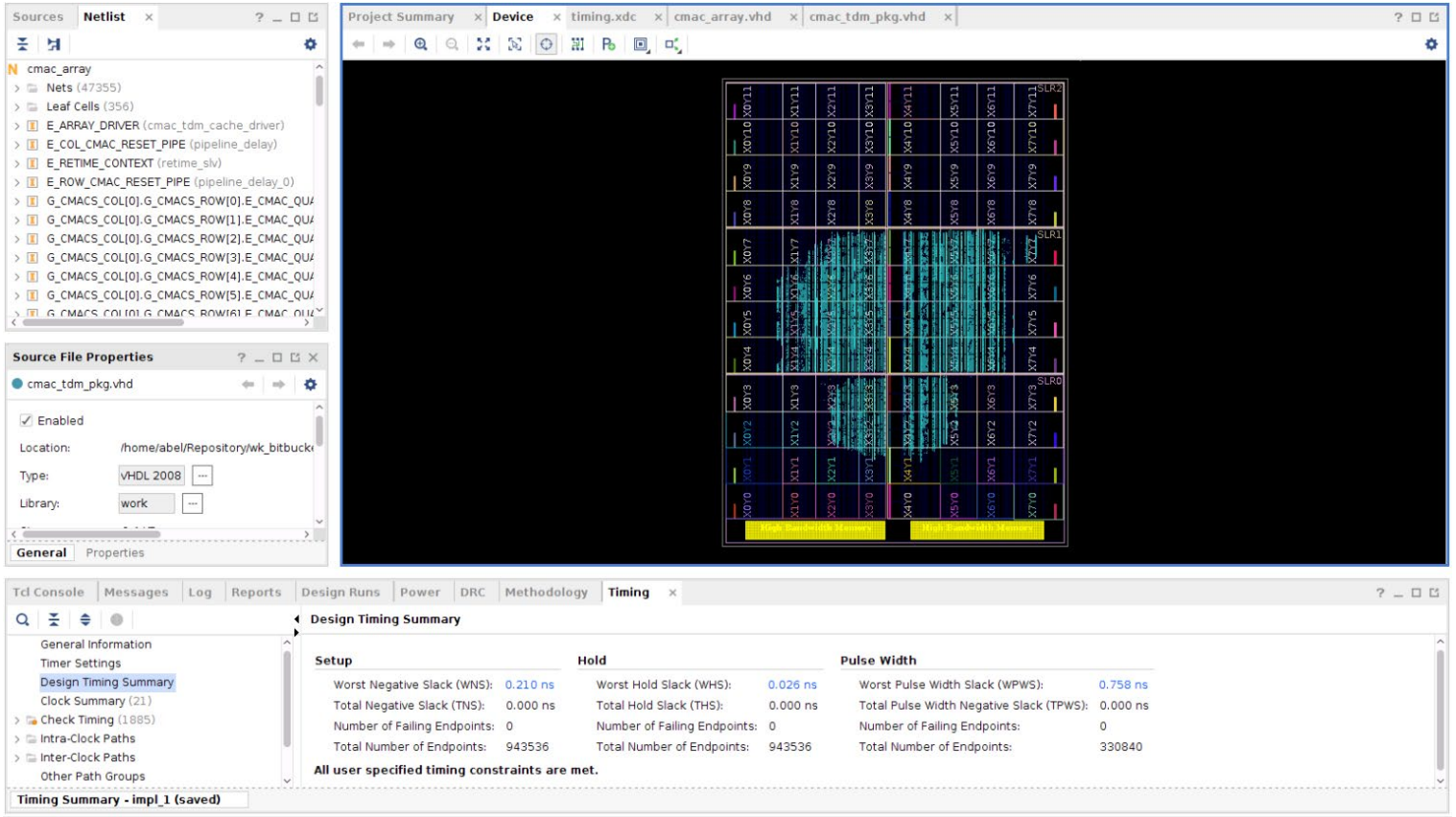
perentie
SKA Low Correlator & Beamformer

TABLE II
IMPLEMENTATION RESULTS FOR XILINX ULTRASCALE+

Style	Width	LUTs	FFs	DSPs	Fmax
optimised	9-bit	86	54	2	640 MHz
inferred	9-bit	99	163	2	450 MHz

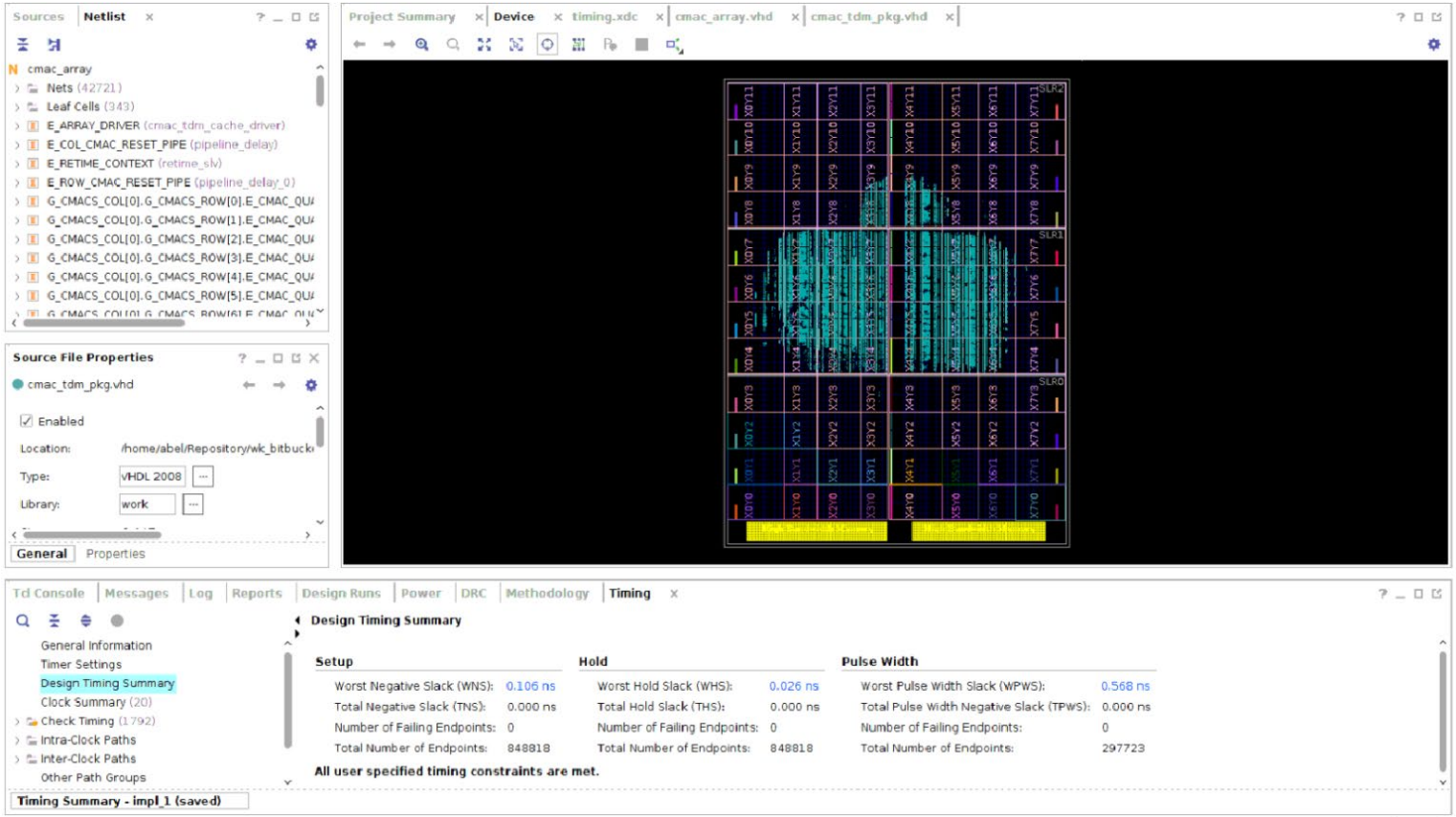
Results: M=19, f=384 MHz

perentie
SKA Low Correlator & Beamformer



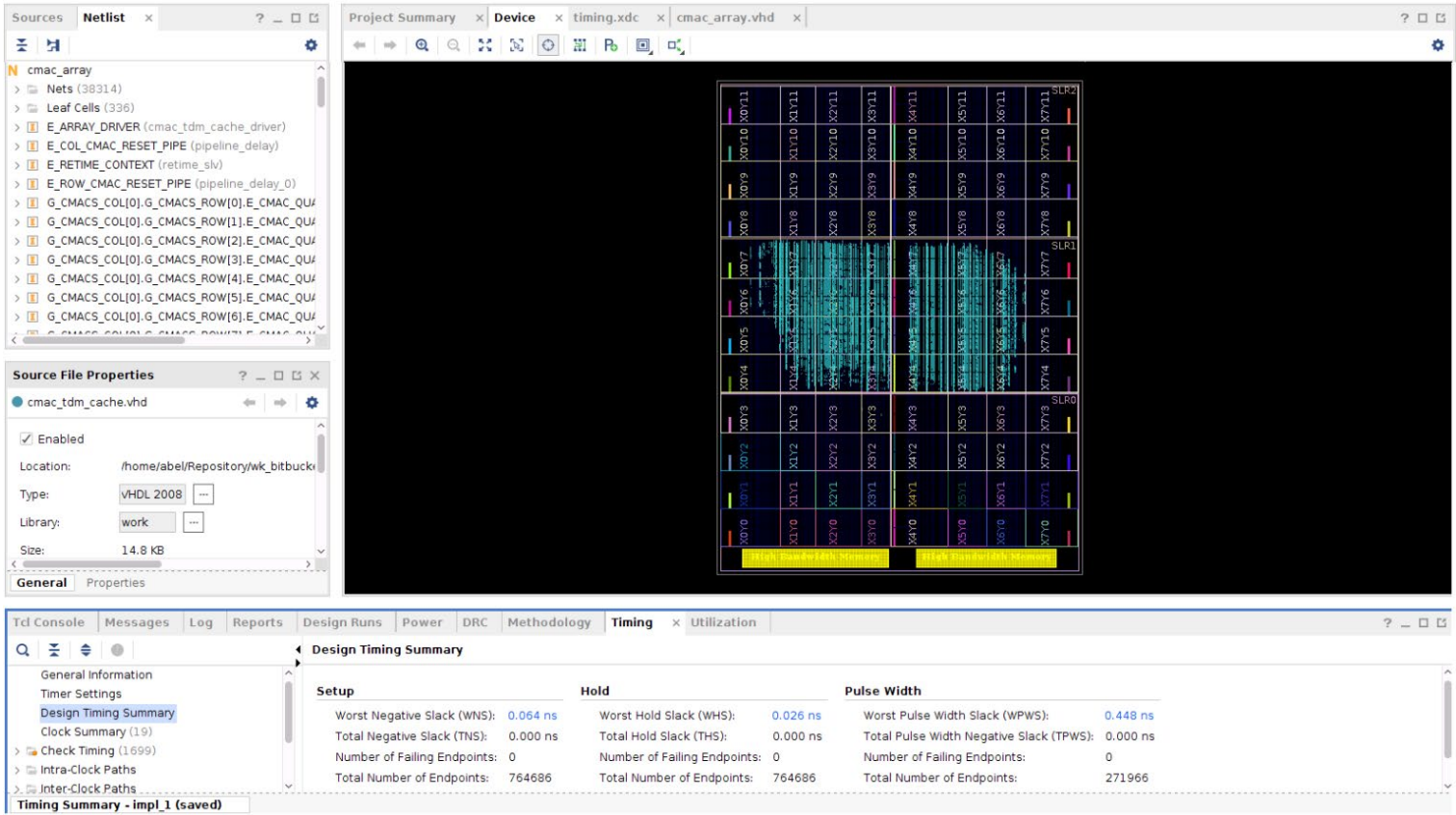
Results: M=18, f=440 MHz

perentie
SKA Low Correlator & Beamformer



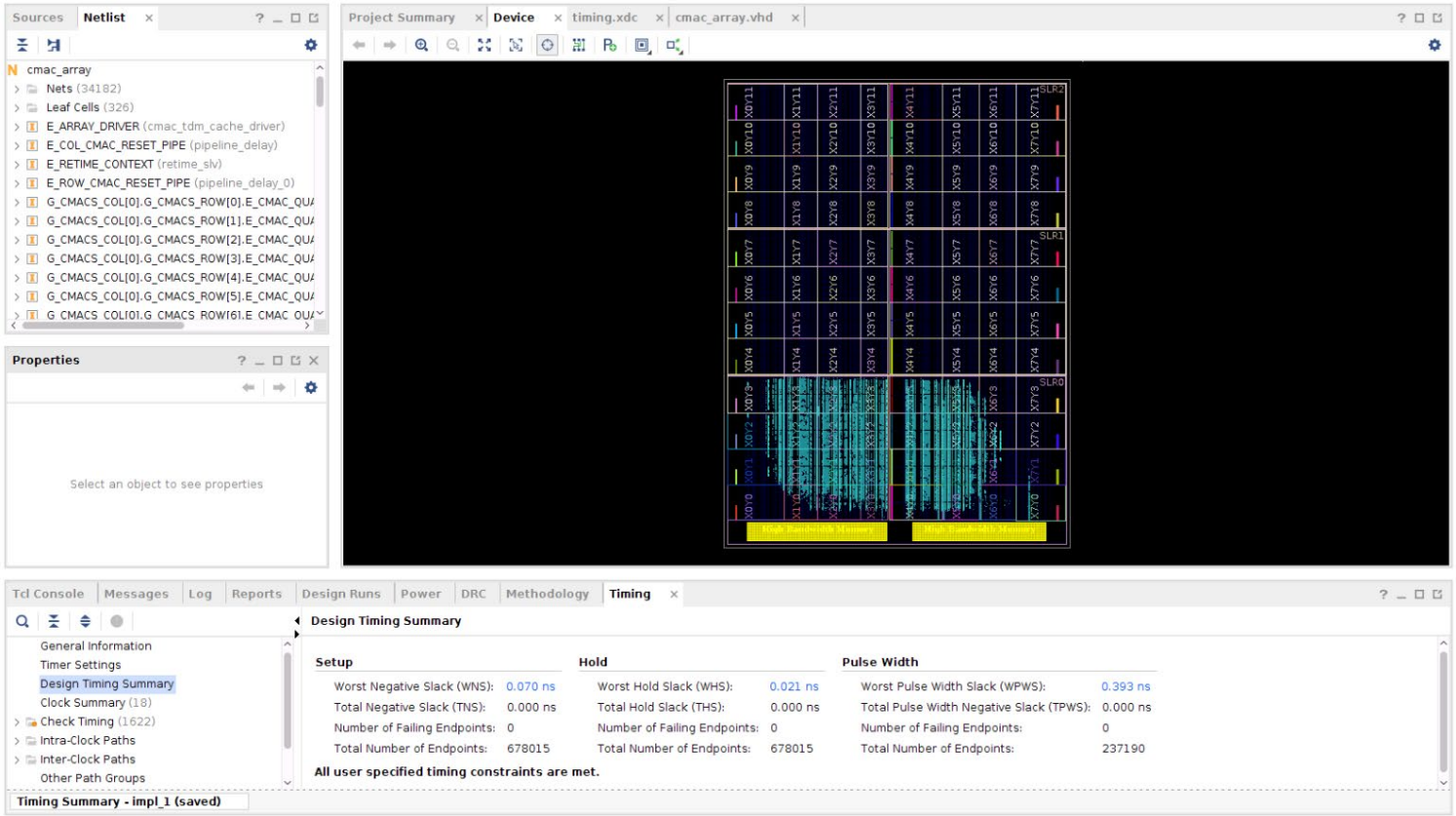
Results: M=17, f=505 MHz

perentie
SKA Low Correlator & Beamformer



Results: M=16, f=534 MHz

perentie
SKA Low Correlator & Beamformer



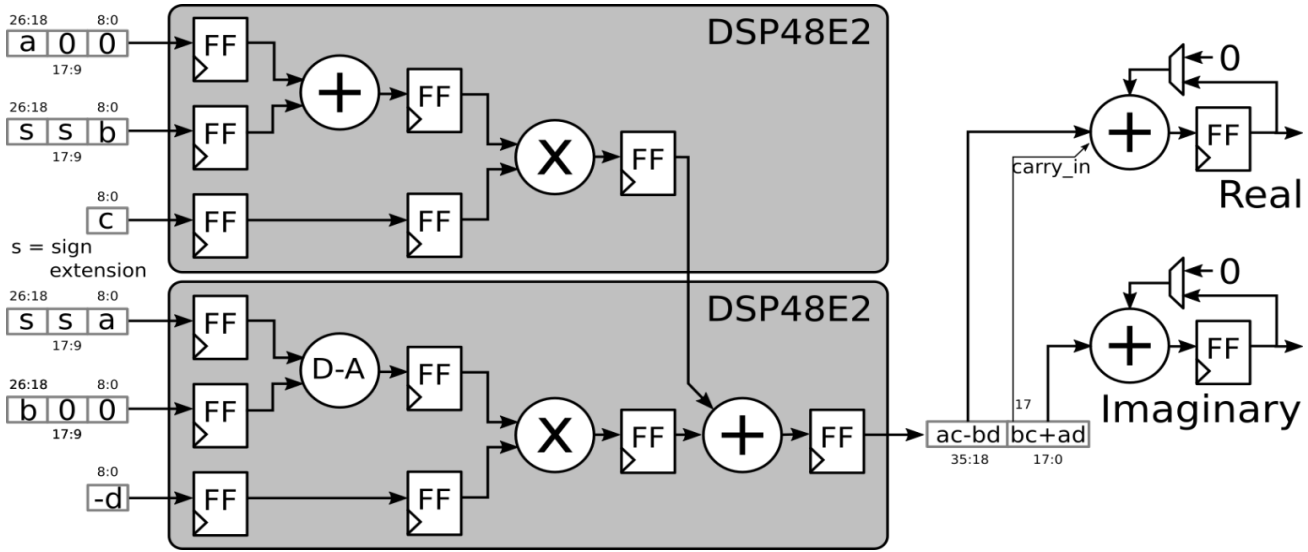
Results

perentie
SKA Low Correlator & Beamformer

Matrix Size (M)	Stations per Row (D)	TDM Slots	Stations Supported (>N/2)	TDM Cache BRAM36s	CMACs	DSP48s	Required Fmax
30 (15 Stations)	35	614	525	150	930	1860	639.6 MHz
32 (16 Stations)	***32***	***512***	***512***	***160***	***1056***	***2112***	***533.4 MHz***
34 (17 Stations)	31	482	527	136	1190	2380	502.1 MHz
36 (18 Stations)	29	422	522	144	1332	2664	439.6 MHZ
38 (19 Stations)	27	366	513	152	1482	2964	381.25 MHz
40 (20 Stations)	26	338	520	160	1640	3280	352.1 MHz

Factor 1/2 of Karatsuba

Factor 2/3 of Karatsuba



Questions / Discussion?

Thank-you!